FATIGUE STRENGTH OF SHAFTS

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INTRODUCTION

At the present time, for a calculation of the fatigue strength of articles in machine building use is generally made of the Miner rule of the summation of the damages and empirical curves of the Weller type. Under these circumstances no account is taken of the gradual development of fatigue cracks during operation, leading to the total failure of the construction. Therefore, methods for calculating the fatigue strength which take account of the growth of fatigue cracks are physically better justified. A method based on a theory of the growth of fatigue cracks developed by one of the present authors [1] is proposed below for calculating the fatigue strength. The actual problem of the fracture of a round shaft subjected of pure bending or torsion is considered. This problem is of great practical importance. The basic assumptions are first formulated and the stress-concentration factors at the edges of a fatigue crack are then determined; a simple differential equation is used to determine the number of cycles up to fracture (the service life of a shaft). Methods are indicated for evaluating the length of the initial crack and the constants of the material, figxring in the theory of fatigue cracks; a numerical example of the calculation of the fatigue service life of a shaft is given.

Basic Assumptions and Statement of Problem

Let a solid cylindrical shaft of round transverse cross section be subjected to pure bending under the action of the bending moment M, rotating with a constant angular velocity ω_0. The fracture of such a shaft takes place as the result of the development of a transverse fatigue crack. The observed forms of these cracks are, as a rule, asymmetric, as a result of both the asymmetry of the initial cracks as well as the instability of the axisymmetric form of the crack with respect to small random changes in the circular line of the front [1]. (For a classification of the structure of fatigue cracks see, for example, [2]). Nevertheless, in the present investigation we shall assume that a fatigue crack at any given moment of time has the form of a circular concentric ring, growing with increasing distance from the boundray of the shaft. Another assumption consists in the fact that the width of the ring at the initial moment of time is equal to l_0, far less than the radius of the shaft.

With loading by a rotating moment, at any fixed moment of time part of the ring is a zone D of instantaneous contact (superposition) of the opposite edges of the fatigue crack, while the remaining part, zone B, is an ordinary open crack of normal type (Fig. 1a). The unfractured part of the transverse cross section (outside of the annular crack and its plane) is denoted by E.

At the common boundary between regions D and B the stress-concentration factor will be assumed equal to zero. This assumption is traditional with respect to theory of contact problems in the theory of elasticity. At the common boundary between regions D and E the stress-concentration factor is greater than zero; this factor is the greater, the closer the investigated point of the contour of the crack to the point A, lying on the line of symmetry of the regions D and E, i.e., the y axis (Fig. 1a). We take the origin of a Cartesian rectangular system of coordinates xy at the center of the transverse cross section of the shaft.

In the zone D + E of the cross section, by virtue of symmetry the tangential stresses are equal to zero; only the axial stress σ_z differs from zero (in the zone B the value of σ_z is also equal to zero). We make the following assumptions:

a) The x axis is a neutral axis, separating the region of compression $y < 0$ and the region of elongation $y > 0$ (i.e., for $y = 0$, $\sigma_z = 0$);

b) the common boundary of regions B and D lies on the x axis.

The second assumption flows out of the first. An idealized form of the transverse cross section under consideration is shown in Fig. 1b.

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2. Stress-Concentration Factor

By K_I we denote the stress-concentration factor at the contour of a crack of normal form (by virtue of the condition of symmetry, the value of K_I depends only on y). By virtue of the assumptions made, the function $K_I(y)$ increases monotonically with a rise in y: from zero for y = 0 to $K_{I\ max}$ at the point A (Fig. 1b).

We approximate the stress σ_z in the region D + E (where it differs from zero) by the following expressions:

$$\sigma_z(x, y) = \begin{cases} \dfrac{by\sqrt{(R-ml)^2-(x^2+y^2)}}{\sqrt{(R-l)^2-(x^2+y^2)}} & \text{for} \quad y > 0, \\ ay & \text{for} \quad y < 0 \end{cases} \tag{2.1}$$

$$(m = 0.3711187875),$$

where R is the radius of the shaft; l is the width of an annular fatigue crack at the moment of time under consideration; and a and b are unknown positive constants, subject to determination.

The approximation (2.1) correctly describes all the principal qualitative special characteristics of the field of σ_z: for y = 0, $\sigma_z = 0$; with a rise in $|y|$, the value of σ_z rises monotonically, $\sigma_z(x, y) = \sigma_z(-x, y)$; and at the common boundary between the regions B and E, the stress has the required singularity. We determine the constants a and b from the two equilibrium equations

$$\iint_{D+E} \sigma_z(x, y)\,dxdy = 0, \quad \iint_{D+E} y\sigma_z(x, y)\,dxdy = M. \tag{2.2}$$

Substituting expression (2.1) for σ_z into (2.2), after certain computations we obtain

$$b = \frac{24M}{R^4[16kqQ(k) + 3\pi P(k)]q^3}, \quad a = bq^3P(k), \tag{2.3}$$

where

$$P(k) = (1-k^2)F\left(k, \frac{\pi}{2}\right) + (2k^2-1)E\left(k, \frac{\pi}{2}\right);$$

$$Q(k) = \int_0^1 \sqrt{1-k^2t^2}\left[(1-k^2)F\left(k\sqrt{\frac{1-t^2}{1-k^2t^2}}, \frac{\pi}{2}\right) + \right.$$

$$\left. + (2k^2-1-k^2t^2)E\left(k\sqrt{\frac{1-t^2}{1-k^2t^2}}, \frac{\pi}{2}\right)\right]dt; \quad k = \frac{1}{q}\left(1-\frac{l}{R}\right), \quad q = 1 - m\frac{l}{R}.$$

Here F(k, π/2) and E(K, π/2) are total elliptical integrals of the first and second kinds, respectively,

$$F\left(k, \frac{\pi}{2}\right) = \int_0^{\pi/2} \frac{d\psi}{\sqrt{1-k^2\sin^2\psi}}, \quad E\left(k, \frac{\pi}{2}\right) = \int_0^{\pi/2} \sqrt{1-k^2\sin^2\psi}\,d\psi.$$

Specifically, for l = 0 we have

$$a = 4M/\pi R^4, \; b = a.$$

We calculate the stress-concentration factor $K_{I\ max}$ at the point A, defined as follows:

$$K_{I\max} = \lim_{\varepsilon\to 0}\left[\sqrt{2\pi\varepsilon}\,\sigma_z(0, y)\right]$$

(ε is the distance to point A).

Using formulas (2.1) and (2.3), we find

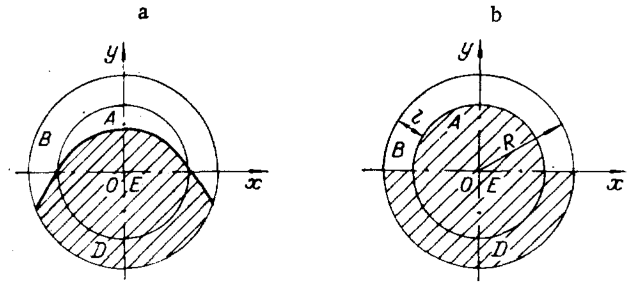

Fig. 1

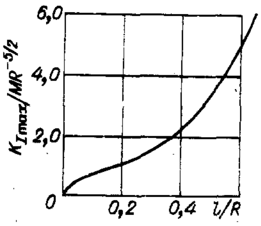

Fig. 2

$$K_{I\max} = \frac{24M\sqrt{\pi kq(1+k)(1-km)}}{R^3q^2[16kqQ(k)+3\pi P(k)]}\sqrt{l}. \tag{2.4}$$

A curve of the dependence of the dimensionless quantity $K_{I\max} R^{5/2}/M$ on the dimensionless depth of the crack l/R is given in Fig. 2.

We note that for very shallow cracks, in accordance with formula (2.4), we have

$$K_{I\max} \sim \frac{4.486M}{\pi R^3}\sqrt{\pi l} \quad \text{as} \quad \frac{l}{R} \to 0.$$

This asymptotic formula corresponds to the uniform elongation of a half plane with an edge crack; the value of the elongating stress at infinity is equal to $4M/(\pi R^3)$ (i.e., to a maximal elongational stress σ_z in a round rod, bent by the moment M). Thus, formula (2.4) is exact for $l \ll R$. The region of small lengths of the cracks makes the greatest contribution to the fatigue life; therefore, precisely in this region an effort must be made to obtain the most exactness of the determination of the stress-concentration factor.

With an error of 1%, formula (2.4) is approximated by the following expression:

$$K_{I\max} = \frac{M}{R^{5/2}}\left[4.486\sqrt{\frac{l}{\pi R}} + \frac{32}{3\pi\left(1-\frac{l}{R}\right)\sqrt{\pi\left(1-\frac{l}{R}\right)}}\frac{l}{R} - 3.075\frac{l}{R} + 1.16\left(\frac{l}{R}\right)^2 + 0.8\frac{0.35-\frac{l}{R}}{\sqrt{1-\frac{l}{R}}}\left(\frac{l}{R}\right)^3\right] \quad \left(0 < \frac{l}{R} < 1\right).$$

3. Determination of Number of Cycles up to Fracture

The rate of growth of fatigue cracks, taking account of plastic and transient kinetic effects, is given by the expression [1]

$$\frac{dl}{dN} = f(K_{I\max}, K_{I\min}), \tag{3.1}$$

where

$$f(K_{I\max}, K_{I\min}) = -\beta\left(\frac{K_{I\max}^2 - K_{I\min}^2}{K_{Ic}^2} + \ln\frac{K_{Ic}^2 - K_{I\max}^2}{K_{Ic}^2 - K_{I\min}^2}\right) + \frac{2\pi}{\omega}v_0\exp\left[\frac{\lambda}{2}(K_{I\max}+K_{I\min})\right]I_0\left[\frac{\lambda}{2}(K_{I\max}-K_{I\min})\right].$$

Here N is the number of cycles of sinusoidal loading; K_{Ic} is the fracture viscosity; $I_0(x)$ is a Bessel function of zero order of an imaginary argument; ω is the frequency of the loading and β, λ, and v_0 are constants of the material.

The time t is obviously expressed in terms of N and ω by the following formula:

$$t = 2\pi N/\omega.$$

Formula (3.1) exhibits good agreement with the experimental data for many materials [1].

Solution of Eq. (3.1) in each actual case makes it possible to take account of the effect of the following factors: the amplitude and the mean value of the load for a cycle, the geometry of the body, and, above all, the value and the arrangement of the initial defect l_0, the frequency ω, the temperature of the body, etc.

The solution of the differential equation (3.1) can be written in explicit form:

$$N = \int_{l_0}^{l} \frac{dl}{f(K_{I\max}, K_{I\min})} \quad (\text{for} \quad l = l_0, \quad N = 0). \tag{3.2}$$

The growth of a fatigue crack continues until the stress-concentration factor attains a limiting value K_{Ic}, after which the fracture will be dynamically unstable. Therefore, if, as an upper integration limit in (3.2), we substitute the critical malue of $l = l_*$, determined as the smallest root of the equation

$$K_{I\max}(l_*) = K_{Ic}, \tag{3.3}$$

formula (3.2) will give the number of loading cycles N_f required for fracture of the construction.

In the problem under consideration, $K_{I\min} = 0$, and $K_{I\max}$ is given by formula (2.4).

We give some results of computations carried out neglecting kinetic transient effects† [here, we must set $v_0 = 0$ in formula (3.2)]. In this case, in accordance with (2.4), formula (3.2) assumes the form

$$\frac{\beta N_f}{R} \int_{l_*/R}^{l_0/R} \frac{dt}{\gamma(t) + \ln[1 - \gamma(t)]}, \tag{3.4}$$

where

$$\gamma(t) = \frac{576\pi(1-m)M^2 t(1-t)[2-t(1+m)]}{R^5 K_{Ic}^2 (1-mt)^6 \left[16(1-t)Q\left(\frac{1-t}{1-mt}\right) + 3\pi P\left(\frac{1-t}{1-mt}\right)\right]^2}.$$

According to formulas (2.4) and (3.3), the critical value of $l_{**} = l_*/R$ depends on the dimensionless moment M_*,

$$M_* = M/K_{Ic}R^{5/2}.$$

Figure 3 shows this dependence $l_{**}(M_*)$, determined numerically using Eqs. (2.4) and (3.3). The curve of Fig. 3 corresponds to brittle fracture; in its physical sense; it is analogous to the Griffiths curve.

It follows from formula (3.4) that the dimensionless service life $N_* = \beta N_f/R$ depends only on l_0/R and M_*. The function $N_*(M_*, l_0/R)$ was tabulated on an M-222 digital computer; Table 1 gives values of this function for a number of values of M_* and l_0/R of practical importance. For the intermediate values, interpolation must be used.

4. Determination of the Constants K_{Ic}, l_0, and β.

To use the result of the theoretical calculations of the service life given in Table 1, the constants of the material of the shaft K_{Ic}, l_0, and β must be known previously.

Evaluation of l_0. A broad literature on the experimental mechanics of fracture and defectoscopy has been devoted to determination of the fracture viscosity K_{Ic} and the initial size of the defect l_0. We take note of only one simple method, which enables a theoretical evaluation of the value of l_0 from the known tensile strength σ_v and the fracture viscosity K_{Ic}.

†For metals, in many practically important cases, they can be neglected.

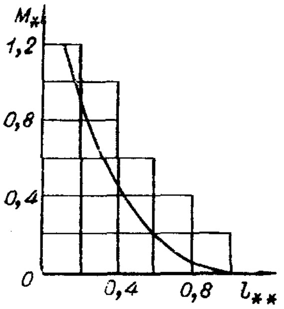

Fig. 3

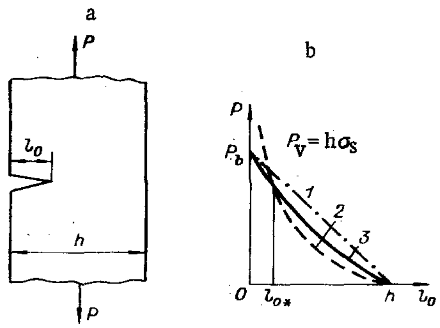

Fig. 4

In our case, for very small lengths of the crack $l/R \ll 1$, the stress-concentration factor at the point A (see Fig. 1b) is equal to $1.1215\sigma\sqrt{\pi l}$, where σ is the greatest stress in a stretched filament at the surface of the shaft. We shall assume that for $\sigma = \sigma_V$ and $K_I = K_{Ic}$, $l = l_0$ (this corresponds to the assumption of the ideally brittle fracture of the shaft). From this, it follows that, in the given case

$$l_0 = 0.253\, K_{Ic}^2/\sigma_V^2. \tag{4.1}$$

It is important to be able to use this simple and important evaluation of the length of a crack in the case of a viscous or transitional type of fracture. Let us examine this question using the example of a plate of width h and constant thickness containing a through edge slit of length l_0 and elongated by the force P at infinity (Fig. 4). The material of the plate is assumed to be ideally elastoplastic with a characteristic tensile strength σ_s. The character of the fracture (brittle or viscous) is determined by the brittleness numbers χ [1] (in the given case there will be two such numbers):

$$\chi_1 = K_{Ic}^2/\sigma_s^2 l_0, \quad \chi_2 = K_{Ic}^2/\sigma_s^2 h.$$

Ideally viscous fracture corresponds to the limiting case where $\chi_2 \gg 1$; in this case the resistance of the plate is directly proportional to the area of the net cross section, and the dependence of the fracture stress P on l_0 will be rectilinear, so that for $l_0 = 0$ $P = h\sigma_s$ while for $l_0 = h$, $P = 0$ (line 1 in Fig. 4b). Ideally brittle fracture corresponds to the limiting case where $\chi_1 \ll 1$; in this case, the dependence of P on l_0 is shown by line 2 in Fig. 4b (it is determined by the methods of the linear mechanics of fracture from a purely elastic calculation). In practically all cases encountered, fracture takes place in accordance with curve 3 of Fig. 4b, lying between the above limiting curves 1 and 2. By l_{0*} we mean the length which corresponds to the intersection of curves 2 and 3. It is obvious that if $l_0 > l_{0*}$, formula (4.1) gives an evaluation from below, i.e., a size of the actual defect greater than that determined by the formula (4.1). If $l_0 < l_{0*}$, formula (4.1) gives an evaluation from above, i.e., a size of the actual defect less than that determined using formula (4.1). For $l_0 = l_{0*}$, the evaluation (4.1) will be exact.

All of the qualitative considerations advanced, relating to the example of Fig. 4, also relate directly to the starting problem (a shaft with an annular crack) with the simple replacement of h by R. In practice, it is generally a case of very small initial cracks, where $l_0 < l_{0*}$; consequently, formulas (4.1), as a rule, give an evaluation from above for the initial cracks. Thus, the use of the value of l_0 determined by formula (4.1) in a calculation of the fatigue life leads to an increased margin of strength, so that the number of cycles up to fracture N_f obtained from such a calculation, other conditions being equal, will be less than the true value.

On the basis of some literature data on σ_V and K_{Ic}, by using (4.1), Table 2 was set up, giving an evaluation from above of the initial length l_0 for several brands of steels.

Evaluation of β. In the case under consideration, $v_0 = 0$, the value of β can be obtained by a comparison between the experimental diagram of $dl/dN \sim K_{It}$ (recorded with $K_{I\,min} = 0$) and the theoretical dependence†

$$\frac{dl}{dN} = -\beta\left[K_{It}^2 + \ln\left(1 - K_{It}^2\right)\right], \quad K_{It} = \frac{K_{I\,max}}{K_{Ic}}, \tag{4.2}$$

which is a partial case of (3.1). The results of such a comparison for a number of materials are given in [1].

†This formula holds when $K_{I\,max} > K_{IY}$, where K_{IY} is the threshold stress-concentration factor [1]. For $K_{I\,min} < K_{IY}$, $dl/dN = 0$.

TABLE 1

l_0/R \ M_*	0,175	0,15	0,1	0,075	0,05	0,03	0,025	0,02	0,015	0,01
0,7645	0,112	0,220	0,713	1,28	2,475	4,879	6,066	7,83	10,71	16,3
0,7265	0,167	0,309	0,935	1,647	3,135	6,109	7,574	9,74	13,3	20,2
0,6903	0,240	0,423	1,216	2,106	3,952	7,629	9,437	12,1	16,5	25,0
0,6560	0,335	0,570	1,569	2,680	4,970	9,512	11,740	15,0	20,4	30,9
0,6232	0,457	0,757	2,013	3,396	6,237	11,85	14,594	18,6	25,3	38,2
0,5919	0,615	0,995	2,572	4,294	7,817	14,75	18,14	23,1	31,3	47,2
0,5620	0,817	1,30	3,276	5,42	9,793	18,37	22,55	28,7	38,8	58,3
0,5333	1,076	1,68	4,160	6,84	12,26	22,89	28,07	35,7	48,1	72,3
0,5058	1,407	2,18	5,28	8,62	15,37	28,56	34,97	44,4	59,8	89,7
0,4795	1,83	2,80	6,70	10,87	19,28	35,68	43,64	55,4	74,5	111
0,4542	2,38	3,61	8,50	13,72	24,22	44,65	54,57	69,2	92,9	139
0,4299	3,08	4,63	10,79	17,3	30,49	56,01	68,4	86,6	116	174
0,4066	3,98	5,94	13,71	21,9	38,46	70,44	85,9	108,8	146	218
0,3843	5,14	7,63	17,46	27,8	48,63	88,84	108,3	137,0	184	274
0,3628	6,64	9,81	22,28	35,4	61,67	112,4	137,0	173,1	232	345
0,3422	8,58	12,6	28,50	45,2	78,47	142,7	173,8	219,5	294	437
0,3224	11,12	16,3	36,56	57,8	100,2	181,9	221,4	279,5	374	556
0,3033	14,44	21,1	47,08	74,3	128,5	232,8	283,3	357,4	478	711
0,2851	18,81	27,4	60,86	95,9	165,5	299,4	364,2	459,2	613	912
0,2676	24,59	35,7	79,03	124,3	214,2	386,9	470,5	593,0	792	1177
0,2508	32,3	46,8	103,1	161,9	278,6	502,9	611,1	770,1	1028	1527
0,2347	42,6	61,6	135,3	212,2	364,6	657,3	798,7	1006	1343	1994
0,2193	56,4	81,5	178,6	279,7	480,2	864,8	1050,5	1323	1765	2620
0,2045	75,3	108,6	237,3	371,3	636,7	1145,8	1391,5	1752	2337	3467
0,1903	101,1	145,7	317,7	496,5	850,7	1529,7	1857,3	2338	3117	4625
0,1767	136,9	197,0	428,7	669,4	1145,9	2059,3	2499,3	3146	4194	6221
0,1638	186,8	268,6	583,5	910,6	1557,7	2797,7	3395,6	4273	5695	8445
0,1514	257,4	369,7	802,1	1250,9	2138,5	3838,7	4658,6	5861	6943	11580

TABLE 2

Material	σ_v, kg/mm^2	K_{Ic}, kg/mm$^{3/2}$	l_0, mm by (4.1)	Source
20KhGSNMA	157	339	1,17956	[3]
St. 3	51	70	0,47662	[4]
16GNMA	57	405	12,77264	[5]
22K	51	300	8,75432	[5]
ShKh15	241	65	0,01840	[6]
50Kh	235	82	0,03080	[6]
50KhN	230	76	0,02762	[6]
A216SS	50	550	30,613	[7]
E24(type 40KhNM)	203	126	0,09747	[8]

TABLE 3

Brand of steel	$\sigma_{0,2}$, kg/mm^2	σ_v, kg/mm^2	δ,%	ψ,%	K_{Ic}, kg/mm$^{3/2}$
15Kh2MFA	53	70,5	20,0	69,4	528
St. 20	24,2	45,8	26,6	57,6	574

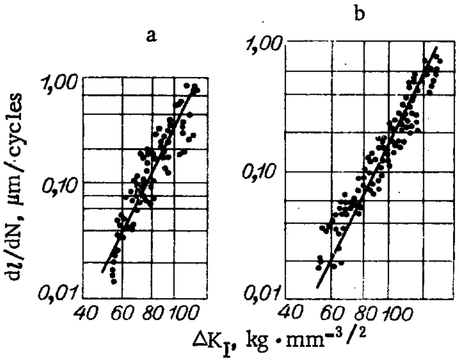

Fig. 5

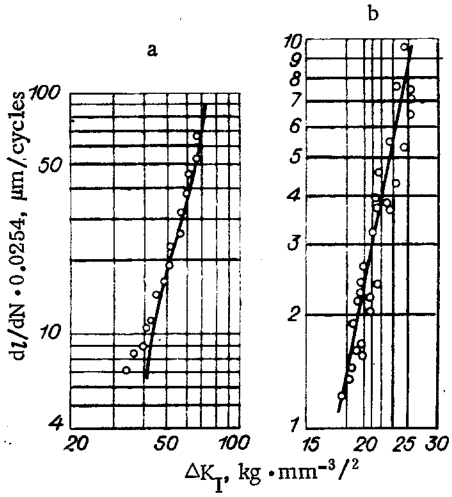

Fig. 6

Using this method, from known experimental data we determine the value of β for some brands of steel used as a material for shafts.

In [9], an investigation was made of the rate of growth of fatigue cracks in low-strength steels 15Kh2MFA and St. 20. The mechanical properties of these steels at room temperature are given in Table 3. In [9], standard solid samples with a thickness of 20 mm were subjected to off-center cyclic elongation (the initial length of the crack was 25 mm). After the growth of a considerable fatigue crack, the same samples were divided for determination of the critical opening at the end of the crack; the fracture viscosity, given in Table 3, was found by a recalculation with respect to the critical opening [10]. The mean factor asymmetry of the loading cycle, i.e., $K_{I\,min}/K_{I\,max}$, was equal to approximately 0.2 (in passing, with sufficiently small factors, their effect is insignificant). Figure 5 gives experimental data of [9] and theoretical curves plotted using (3.1) (where $v_0 = 0$ and $K_{I\,min}/K_{I\,max} = 0.2$) for the following values of β:St. 20 (Fig. 5a), $\beta = 0.234$ mm; steel 15Kh2MFA (Fig. 5b), $\beta = 0.0823$ mm. As can be seen, experiment and theory are in excellent agreement with the above values of β.

TABLE 4

Material	$\sigma_{0.2}$, kg/mm²	σ_V, kg/mm²	K_{Ic}, kg/mm³/²	β, mm	l_0, mm	Source
Steel 20	24,2	45,8	574	0,234	39,739	[9, 10]
15Kh2MFA	53	70,5	528	0,0823	14,191	[9, 10]
NU-80 (type 15KhN)	76	89	354	0,012	4,003	[11]
Ferrite steel	33,1	44,8	141,6	0,001524	2,527	[12]

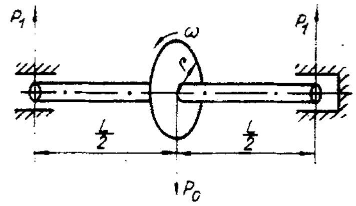

Fig. 7

In [11], an investigation was made of the growth of cracks with low-cycle fatigue in domestic steel 15KhN with a martensite structure ($\sigma_{0.2}$ = 76 kg/mm², σ_V = 89 kg/mm²). Flat samples (measuring 22.86 mm × 152.4 mm × 760 mm) with a central surface crack were subjected to cyclic loading by an axial force. The maximal level of the nominal stresses with cyclic loading was 50-60% of the yield point. The initial crack had the form of a half circle of radius 2.54 mm with its center at the surface of the sample; it was assumed that during the process of growth of the fatigue crack its form remained semicircular with the same center.

The asymmetry factor of the cycle was equal to zero. Figure 6a gives a theoretical curve plotted using formula (4.2) for the following values of the constants:

$$\beta = 0.012 \text{ mm},$$

$$K_{Ic} = 354 \text{ kg/mm}^{3/2}.$$

Figure 6 also gives the experimental data of [11]. The agreement between the two sets of data was very good.

In [12], an investigation was made of the rate of growth of fatigue cracks in ferrite steel (%): C, 2.2; Mn, 0.82; Nb, 0.012; P, 0.006; S, 0.021; Ti, 0.05; Ni, 0.05; Cr, 0.05; Mo, 0.05; Cu, 0.05; W, 0.005.

The thickness of the sample was 9.5 mm, the tensile strength was 44.8 kg/mm², and the yield point was 33.1 kg/mm². The asymmetry factor of the cycle was 0.05. Figure 6b shows a theoretical curve, plotted for the following values of the constants:

$$\beta = 0.001524 \text{ mm}, \ K_{Ic} = 141.6 \text{ kg/mm}^{3/2}.$$

The curve well describes the experimental data of [12].

Data obtained at this point for the constants l_0, β, and K_{Ic} are given in Table 4; they can be used in calculations of the fatigue strength of shafts.

5. Concrete Example of Calculation of the Service Life of a Shaft

At the middle of a shaft of length L let there be rotating a heavy flywheel with a weight P_0 (the weight of the shaft is not taken into consideration). By ε we denote the eccentricity of the flywheel (Fig. 7). The maximal value of the thrust reaction P_1 in the bearings

$$P_1 = \frac{1}{2} P_0 \left(1 + \frac{\omega_0^2}{g} \varepsilon\right)$$

(g is the acceleration of gravity).

The greatest value of the bending moment M in the most dangerous (middle) cross section of the shaft

$$M = \frac{1}{4} P_0 L \left(1 + \frac{\omega_0^2}{g} \varepsilon\right).$$

The material of the shaft is steel 15Kh2MFA, whose principal constants are given in Tables 3 and 4.

We assume the following numerical values of the parameters: R = 80 mm, L = 2000 mm, ε = 0.1 mm, ω_0 = 60 rpm, and P_0 = 14,170 kg. Here the elongational stress in a surface filament of the most dangerous cross section is $\sigma_V/4$. Using (3.4), in this case we obtain

$$N = 4 \cdot 10^4 \text{ cycles.}$$

With the use of a steel of type 15KhN for fabrication of the shaft, for the same values of the parameters R, L, ε, and ω_0 ($\sigma_{max} = \sigma_V/4$, P_0 = 17,890 kg), the service life is increased by approximately four times, i.e.,

$$N = 1.7 \cdot 10^5 \text{ cycles.}$$

Interpolation using Table 1 leads to close results.

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